The Impact of a Measurement-Focused Program on Young Children's Number Learning

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Children form mathematical concepts at an early age, and many of these concepts are linked to informal measurement experiences. However, mathematics education at school is often focused on counting and numbers. A mathematics intervention using a measurement-focused program replaced the usual mathematics program for 40 children entering their first year of school. The results of Counting and Place Value interviews held at the beginning and end of the school year are reported. Findings indicate that a "student-active" measurement-focused program can stimulate the development of children's number knowledge; however, additional counting may benefit children's number skill development.

Children acquire considerable mathematical knowledge before they enter school (Clarke, Clarke, & Cheeseman, 2006). This knowledge is built through experience and exploration in meaningful life contexts. Children's lives are rich in problem solving where children make decisions about number, position, and size in authentic measurement contexts (Clements & Sarama, 2011). In contrast, the mathematics that many children experience on entry to school is heavily number-focused (Benz, 2012). Young children have intuitive and informal capabilities in both spatial and geometric concepts, and numeric and quantitative concepts (Bransford, Brown, & Cocking, 1999). As Clements and Sarama (2011) noted, "children must learn to mathematize their informal experiences by abstracting, representing, and elaborating them mathematically" (p. 968). We believed that children could learn number more meaningfully through a measurement-centred curriculum where authentic experiences required children to solve problems. In this study, we sought to answer the question: What is the impact of a mathematics program based on measurement activities on young children's number learning?

Background

The idea of using a measurement-focused curriculum is not new. In Russia, a curriculum developed by Davydov, Gorbov, Mukulina, Savelyeva and Tabachnikova (1999) was based on findings by Davydov (1975). These authors developed a mathematics program without numbers where the physical attributes of objects were described and compared. It was intended that children should develop, through different activities, an understanding of equality and be able to describe comparisons with relational statements. Davydov described a comprehensive theoretical progression of children's thinking about measurement concepts which was implemented and tested in the Hawaiian program *Measure Up* (Dougherty & Zilliox, 2003). The project started with a generalized approach and then applied the knowledge gained to specific cases. The team worked on ways to deliver the theoretical Russian approach in classrooms. The project identified at least six types of instruction: (1) giving information, (2) simultaneous recording, (3) simultaneous demonstration, (4) discussion and debriefing, (5) exploration guided, and (6) exploration unstructured. The order (1-6) represented a continuum from most teacher-active to most student-active. Sophian (2007) maintained that the relationship between measurement concepts and proportionality supported children to develop deep understandings of

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mathematical structures and properties of number. However, the Russian curriculum approach was not universally admired. It was criticised for its abandonment of numbers, its use of letters as variables, and its focus on early abstraction and generalization (Freudenthal, 1974; Steinweg, 2013).

A contrasting view of the relationship between number and measurement was conceived by Steffe (2010), who viewed discrete quantities and continuous quantities as connected. He named his four counting schemes as discrete quantitative measuring schemes and described four stages: (1) the perceptual counting scheme, (2) the figurative counting scheme, (3) the nested number sequence, and (4) the explicitly nested number sequence. For Steffe, the development of awareness of continuous quantities was analogous to his four stages of an awareness for discrete quantities.

Sophian (2007) questioned the common perspective that "children's thinking begins with the premise of counting, or some form of determining the numerical values of discrete quantities, [and] is the foundation for much of children's developing knowledge about mathematics" (p. 3). She described a contrasting position "that what is most fundamental for mathematical development is not counting or other mechanisms for apprehending numerosity, but rather basic ideas about relations between quantities" (p. 3). It is this comparison-of-quantity perspective that informed the present study.

Clements and Sarama (2009) wrote that "measurement can be defined as the process of assigning a number to a magnitude of some attribute of an object, such as its length, relative to a unit. These attributes are continuous quantities" (p. 163). This definition emphasises the links between number and measurement and the commonalities between discrete and continuous quantities. It was used in the present study.

The intended curriculum (Van den Akker, 2003) is documented in the Australian Curriculum: Mathematics (AC:M; Australian Curriculum, Assessment and Reporting Authority [ACARA], 2012), which contains the content strands Number and Algebra, Measurement and Geometry, and Probability and Statistics. The strands are intended to be integrated in practice together with the proficiencies: understanding, fluency, problem-solving, and reasoning. Curriculum outcomes for number and place value in the first year of school (Foundation) focus on counting numbers to 20, counting and subitising small collections, and comparing collections. Measurement outcomes specify "Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning" (ACARA, 2012). While the national curriculum has been implemented since the original comparative data used in this paper were collected, the Victorian curriculum, at the time, specified very similar learning outcomes for students in their first year of school. In this study, we used the Australian Curriculum and investigated a change of emphasis in the implemented curriculum from number to measurement.

Method

A year-long project was conducted with children five to six years of age entering school in Victoria to examine the impact of a mathematics program based on measurement activities on young children's number learning. The implemented curriculum in this experiment emphasised measurement, and de-contextualised number was not taught at all – Number was used to quantify attributes.

Detailed planning was undertaken by two teachers who worked with the 40 children. The program was designed for a school with a commitment to a Reggio Emilia philosophy of education (Rinaldi, 2006) in which the second author (Yianna) worked at the time. The school offered opportunities for children to learn through problem solving, and they were expected to engage with mathematical thinking every day; teachers were expected to interact with children to challenge and extend their thinking. The measurement-based program was characterised as being "student-active" (Dougherty & Zilliox, 2003), where the planned experiences involved unstructured or guided exploration by the children. Daily mathematics focused on measurement tasks and problems. The classroom observations later in this paper illustrate such problems. Teachers created detailed planning documents and observational records were kept of the children's actions and ideas. Teachers regularly met to discuss the mathematical learning of individuals and the group as a whole.

The study was characterised as design research because it was interventionist, iterative, practical in a real context, and process-, utility-, and theory-oriented (Van den Akker, Gravemeijer, McKenney, & Nieveen, 2006). The merit of the program was evaluated by looking at the learning outcomes of the children. We report results of analysis of interview data related to Counting and Place Value together with two classroom observations.

The interview protocol used was developed in the Early Numeracy Research Project (ENRP; Clarke et al., 2002). Space constraints allow only the reporting of Counting and Place Value results. The data from the original large-scale research project are used here as reference points in the analysis of the current data. In the present study, the second author interviewed the children at the start of the year, and an independent trained interviewer conducted the end-of-year interviews. All student responses were independently coded as numerically representing the Growth Points of a theoretical framework according to the coding protocols developed by Clarke and his colleagues.

The mean Growth Point (GP) codes of the children entering school for the first time (five- to six-year-olds) at the beginning and the end of their first year at school were calculated for Counting and Place Value. These means were then compared to the three cohorts of reference school (control) data in the ENRP (Clarke et al., 2002) because for three consecutive years data were collected from a representative range of schools matched to the research schools. Reference schools received no experimental treatment and therefore could be considered to represent children on entry to school and at the end of the first year of school in Victoria. From this large dataset, a matched sample was constructed for Yianna's data. Her school had been a reference school for the original research project. The assessment instrument was identical to that used with Yianna's students, and the interview was conducted by an independent trained interviewer.

Findings

ENRP Growth Points were defined to describe children's developing understanding of each mathematical domain. Here, only the two domains will be used. The Counting Growth Points (Figure 1) described the development of children's counting by ones, as well as by the multiples of two, five, and ten. The Growth Points were concerned with children's production of number name sequences. However, the Growth Points were also concerned with children making the count-to-cardinal transition described by Fuson (1982) as being able to think about the number sequence to solve problems.

The Growth Points in Counting were devised to articulate the key steps taken by children in developing their understanding of the number sequence. However, these Growth Points do not describe children's use of counting in addition, subtraction, multiplication, and division problem solving situations. Such strategies are described in separate domains. Only the relevant growth points in the Number domains will be presented in the analysis of the results.

0. Not apparent: *Not yet able to state the sequence of number names to 20.*
1. Rote counting: *Rote counts the number sequence to at least 20, but is not yet able to reliably count a collection of that size.*
2. Counting collections: *Confidently counts a collection of around 20 objects.*
3. Counting by 1s (forward/backward, including variable starting points; before/after): *Counts forwards and backwards from various starting points between 1 and 100; knows numbers before and after a given number.*
4. Counting from 0 by 2s, 5s, and 10s: *Can count from 0 by 2s, 5s, and 10s to a given target.*
5. Counting from x (where x > 0) by 2s, 5s, and 10s: *Given a non-zero starting point, can count by 2s, 5s, and 10s to a given target.*
6. Extending and applying counting skills: *Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical tasks.*

Figure 1. ENRP Counting Growth Point framework (Clarke et al., 2002).

The mean Growth Point results in Counting and Place Value are presented in Table 1, where the original cohorts for three years are labelled C1, C2, and C3, and Yianna's children are labelled YP. A matched school dataset is reported as MS. These data were collected in the same school over the original three years of the ENRP via a random sampling of the students. The MS students followed the intended curriculum at the time.

Table 1
Comparison of Mean Growth Point Codes for Counting and Place Value

	Counting		Place Value	
	Mar.	Nov.	Mar.	Nov.
C1 (*n* = 438)	0.78	1.80	0.36	0.93
C2 (*n* = 504)	0.86	1.74	0.36	0.98
C3 (*n* = 523)	0.88	1.83	0.34	0.99
YP (*n* = 40)	0.33	0.93	0.34	0.95
MS (*n* = 51)	0.42	1.49	0.29	0.88

An examination of the comparative results (Table 1) reveals that Yianna's children came to school with counting knowledge not as sophisticated as most children involved the original research. On average, Yianna's students began school unable to recite the number names to 20 (mean GP 0.33) and by the end of the year, they had improved their rote counting skills less than the control groups (mean GP 0.93). The mean for the state was almost at Growth Point 2, where the child can reliably count a collection of around 20 objects. By the end of the first year of school, on average, the children in this study could recite the number names to 20 but not reliably count collections of 20 objects.

Matched Group Comparisons in Counting

The results of the experimental group (YP) compared to the matched school data (MS) in Counting (Table 2) show that the two groups began the year with very different percentages of children achieving Growth Points 0-1 Almost three-quarters of YP children

(73%) were unable to say the number name sequence to 20. Only 10% of YP children could count a collection of over 20 objects. Clarke, Clarke, and Cheeseman (2006) reported that 39% of Victorian children achieved this skill. The matched control group had 36% at GP2 or above, indicating that they were more aligned with the broader population on entry to school. The poor counting knowledge on entry to school of Yianna's group was thought by their teachers to be due to a combination of factors: high Language Background Other Than English, high non-attendance at pre-school, low socio-economic background, and a large proportion of newly-arrived migrant families. In part, these factors were a stimulus to trying a different approach to mathematics for the children as a focus on counting had limited success in previous years.

Table 2
The Numbers of Children at Each Growth Point in Counting

	YP Counting		MS Counting	
Growth Point	Mar. *n* = 40 (%)	Nov. *n* = 40 (%)	Mar. *n* = 65 (%)	Nov. *n* = 48 (%)
0	29 (73)	14 (36)	36 (55)	7 (15)
1	7 (17)	15 (38)	6 (9)	6 (13)
2	4 (10)	7 (18)	9 (14)	33 (68)
3	0	2 (5)	14 (22)	1 (2)
4	0	0	0	1 (2)
5	0	1 (3)	0	0

The greatest differences exist between the two groups in the end-of-year results. Yianna's children had learned to count in a measurement-focused curriculum but 14 (36%) remain on GP 0, not yet able to state the number names to 20. A further 15 (38%) could verbally count but were unable to count a collection of objects. Only 10 children could count reliably (26%). The two children who could count forwards and backwards from various starting points between 1 and 100, and who knew numbers before and after a given number, exhibited knowledge described in the AC:M as Year 1 outcomes. One student achieved GP 5, showing the ability to count from a non-zero starting point and to count by twos, fives, and tens to a given target (AC:M Year 2). In contrast, at the end of the year, most children in the control group (MS) could reliably count collections – GP 2 (68%).

Place Value

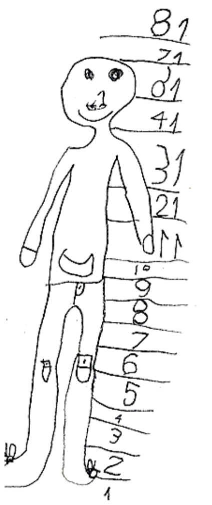
An examination of the beginning and end-of-year data of each of the cohorts for Place Value (Table 1) reveals very similar patterns of results between the three cohorts and the matched sample and the experimental group. By the end of their first year of school, the children interviewed demonstrated that they had a sound understanding of single digit numbers. This relates to GP 1 in the domain of Place Value: Reading, writing, interpreting, and ordering single-digit numbers.

Looking at matched group comparisons in Place Value (Table 3) shows patterns of results in the experimental (YP) and control (MS) groups of Place Value that are very similar. The difference in these data is that by the end of the year, a total of 16% of the children in the measurement-based curriculum group (YP) had extended their knowledge of numbers and the number system beyond single-digit numbers, reading, writing, and interpreting two-digit (GP1) and three-digit numbers (GP2) successfully. Place Value knowledge indicates a developing awareness of the number system as a whole. The “top” (8%) of the experimental group had children who had mastered reading, writing, and interpreting three-digit numbers. This could be attributed to the need to use larger numbers in the measurement context or the removal of a “ceiling effect” of the intended curriculum.

Table 3
The Numbers of Children at Each Growth Point in Place Value

	YP Place Value		MS Place Value	
Growth point	Mar. n = 40 (%)	Nov. n = 40 (%)	Mar. n = 51 (%)	Nov. n = 48 (%)
0	29 (73)	13 (32)	38 (75)	14 (29)
1	10 (25)	21 (52)	13 (25)	29 (60)
2	1 (3)	3 (8)	0	5 (11)
3	0	3 (8)	0	0
4	0	0	0	0
5	0	0	0	0

An examination of the interview results tells only part of the complex story of the intervention. Daily classroom observation data, collected during the measurement-focused program, add to the picture. Two observations about length will illustrate some of the children’s thinking.



Classroom data: Mitchell’s height. In their first week at school, children were invited to consider: How tall are you? How big are your feet? How much do you weigh? These invitations to explore were posted in the mathematics corner of the classroom together with a height chart (in centimetres), scales, Unifx cubes, wooden sticks, assorted blocks, a measuring tape, and a 1 metre ruler.

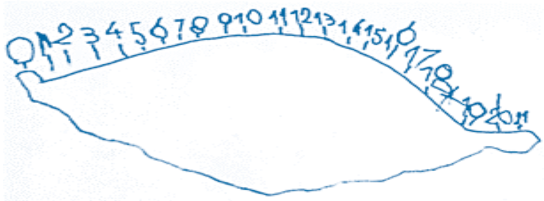
Mitchell (5 years) (pseudonym) was observed sitting, pencil and paper in hand, looking very busy. School had not officially begun for the day, but Mitchell had decided he would to draw himself against a height chart because he said that he already knew how tall he was.

“Look I’m 17 tall, see, not 18, because the line to my head is at 17.” As Mitchell’s drawing shows, on entry to school, Mitchell understood that measuring height uses numbers and he knew the number sequence to 17 with only one number missing. For him, the 17 line matched his height. Despite the absence of a 15 in his number sequence, he drew and labelled spaced intervals to illustrate his understanding that he was 17 tall. His counting began at one, and the origin of his measure is not drawn. The labels show that he understood that the count referred to the length of each interval. He carefully matched the line for 17 to the top of his head in the drawing, showing accuracy and an awareness of the end point of the measure. While it is not clear where the notion of 17 arose, Mitchell had in his mind that he was 17 tall and he could clearly show what this meant to him using an approximation of a number line.

Classroom data: Measuring the autumn leaves. The following conversation between Eliza (pseudonym) and her teacher shows how children’s attention was naturally drawn to salient features of length measurement.

Eliza: "I don't know how much this leaf is. …..It's 300."

Looked at the millimetres on the ruler she had placed along the length of the leaf



T: "Where did you start your measuring?"

Eliza: "Two."

T: "Why two?"

Eliza: "I don't know. I think I should start at zero."

T: "Where is zero on your ruler?"

Eliza pointed to the zero

T: "So, can you start measuring from zero? Does that help?"

Eliza: "Yes."

T: "Can you measure from the start of your leaf?'

Eliza: "Yes. I will draw a line so I know where the end is."

Eliza traced the outline of the leaf

T: "What are you doing with your leaf?"

Eliza: "I'm putting numbers on something."

T: "What are the numbers for?"

Eliza: "Measuring something. Anything – like shelves, chairs, and leaves."

T: "So why are you putting the numbers along the leaf?"

Eliza: "To know how much the leaf is."

Eliza knows the purpose of numbers in quantifying a length although as yet she has no apparent concept of unit. Her awareness of the origin of the measure was raised in conversation, the number zero became a meaning for her in this context.

Discussion

In an attempt to find a way to make mathematics meaningful for children, a measurement-focused program replaced a counting-based program. The results show that learning achievements in Counting were negatively impacted. More than half of the children in the experimental group were not counting collections reliably by the end of their first year of school. This finding suggests that children entering school – at least those with poor English language skills and a low socio-economic background – seem to need explicit counting practice with the sequence of number names to 20 in addition to experiences with number in measurement contexts. Perhaps the theoretical connection between discrete quantities and continuous quantities as described by Steffe's (2010) counting schemes could also be made more explicit to teachers.

The Australian Curriculum: Mathematics has a Number and Place Value strand at Foundation level that lists five outcomes where counting as quantifying is emphasised. While counting is a key to young children's mathematical futures, children need more than repetitive counting routines and learning through skill-driven tasks (Mulligan & Mitchelmore, 2009). The learning contexts of the measurement-focused program offered children many opportunities to use numbers in relevant and meaningful ways. Children's development of a sense of the number system as reflected in the Place Value results.

The genuine measurement context for number stimulated some children to go beyond the intended curriculum (AC:M) in measurement outcomes. These children could use units to measure and quantify, whereas the intended curriculum specifies only the use of direct and indirect comparisons. This study supports the findings of recent studies (Cheeseman, 2012; Macdonald, 2011; McDonough, 2011) that show that young children are capable of more sophisticated concepts of measurement than the AC:M (2012) specifies.

An implication of this study is that measurement contexts can be productively used with young children to stimulate number knowledge and reasoning.

Like Sophian (2007), we question the perspective that mathematical basis of children's thinking is counting or determining the numerosity of discrete quantities. This study supports a comparison-of-quantity perspective described by Sophian as an effective approach for young learners of mathematics. We are not advocating eliminating counting; we are advocating using counting to determine numerosity in measurement contexts. This study has shown that, in principle, when young children measure, they use numbers in meaningful ways and can acquire number competencies.

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